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1 Word Problems: Rate

Like ratios, it's useful to think of rates as fractions. In fact, rates are essentially the same as ratios, with one key difference. Ratios express the relationship between two like things: if the ratio of men to women is 3 : 4, we may separate people into two segments, but they are all people. Rates, on the other hand, express the relationship between two unlike things.

The most common type of rate is speed. You've probably heard the phrase "miles per hour" or "kilometers per hour" so many times in your life you've long since stopped thinking about it as a relationship between two different quantities. But a relationship is, in fact, what it is. When you say you're traveling at 60 miles per hour, you're expressing a relationship between the number of miles you're traveling and the number of hours you're traveling:

$\frac{60 \text{ miles}}{1 \text{ hour}}$

You can handle many of the rate problems you'll see on the GMAT if you treat them just like the ratios you saw a few chapters ago. Just as with ratios, you'll be given a relationship ("60 miles per hour") and one actual number (say, 180 miles). With those, you can set up a ratio:

$\frac{60 \text{ miles}}{1 \text{ hour}} = \frac{180 \text{ miles}}{x \text{ hours}}$

Since the units are the same on both sides of the equation, you can forget about them. Then, cross-multiply and solve for x:

$60x = 180(1)$

$x = \frac{180}{60} = 3$

Rate problems need not be speed-related, though. You will see plenty of rates that express some form of speed, such as:

miles per hour
widgets per day
dollars per month

However, a rate can consist of any relationship of two unlike things. Here are some examples of rates you might come across on GMAT questions:

sales per customer
GDP per capita
shares of stock per portfolio

While you may be most comfortable with speed-related rates, it's important to understand rates at a more abstract level, one that allows you to accomodate these other types of rates with ease. To reach that level, force yourself to express every single rate you see as a fraction, consciously using the word "per" as a signal. Just as "miles per hour" means you'll have some number of miles divided by some number of hours, "GDP per capita" means you'll have some amount of GDP divided by some number of people.

In most such cases, the problem isn't much more complicated than anything you'd see on a ratio question. The techniques discussed in that section work just as well here.

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GMAT rate questions get tricky when the test combines multiple rates. In fact, one type of question, sometimes called "work" or "simultaneous rate," merits the entire next chapter. Before getting to work problems, we'll take a look at a couple of others way in which the GMAT will combine rates.

The first type of problem is "average rate" or "average speed." An example might go like this:

> "Karen drove 100 miles at a speed of 40 miles per hour, then another 100 miles at a speed of 50 miles per hour. What was her average speed for the entire 200-mile trip?"

Most people, upon initial exposure to this type of question, have a reflex answer: 45 miles per hour. Accordingly, most people are wrong. To discover why, we'll need to walk through the problem, step by step.

The formula for average speed is just like that for any other type of rate. If we're looking for miles per hour, we need to find the total miles over the total number of hours. Total miles in this case is easy: 200 miles. Total hours takes more effort. 100 miles at 40 miles per hour takes 2.5 hours, while 100 miles at 50 miles per hour takes 2 hours. Thus, total hours is 4.5, and the average speed is:

$$\frac{200}{4.5} = 44\frac{4}{9}$$

Why do we get such a counter-intuitive answer? The key is the times that we solved for. Karen spent more time driving at the slower speed, so her average speed was closer to 40 than 50. The average speed is weighted by amounts of time, so the fact that the distances were the same doesn't mean the weights are equal. (For more on weighted averages, consult the chapter specifically covering that topic.)

It's great if you understand exactly why the intuitive answer is wrong. But most importantly, you need to remember that to solve for an average speed, you must calculate total distance and total time, as we did working through this example.

The other type of combined rate problem involves two objects moving toward each other. (Or, in another variation, one catching up with the other.) The stereotypical example goes something like this:

> "Stations A and B are 100 miles apart. If Train X leaves Station A moving at 20 miles per hour and Train Y leaves Station B at the same time moving at 30 miles per hour along a parallel track, how long will the trains travel before they meet?"

The key concept needed to solve a problem like this is that the trains move toward each other at the sum of their rates. After one hour, Train X will have traveled 20 miles, and Train Y will have traveled 30 miles. So after that first hour, the trains are 50 miles closer to each other than they were when they started. Another way to put that is to say that the trains are converging at 50 miles per hour.

The math involved is very simple. In fact, it's the same math you'd need to figure out how long it takes you to drive 100 miles at a speed of 50 miles per hour. It's 2 hours. If the question asked, instead, how far Train Y traveled

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before the trains meet, you would begin the problem the same way. Once you discovered that it took 2 hours before they met, you'd use Train Y's speed (30 miles per hour) to determine how far Train Y traveled in 2 hours: 60 miles.

The variation–one object catching up with another–is very similar. Here's an example of the basic framework that these questions follow:

> "Ron and Sara are driving along the same road. Sara is driving at a constant rate of 30 miles per hour, while Ron is driving at a constant rate of 40 miles per hour. If Ron is currently 5 miles behind Sara, at what point will Ron catch up with Sara?"

Much like the converging trains of the previous example, the first step here is to distill the two rates into a single number. We don't particularly care how far Ron or Sara get, or how fast they do so. What matters is how fast Ron catches up. We can find that by subtracting Sara's rate from Ron's rate, for a "catch-up rate" of 10 miles per hour. In an hour, Ron drives 10 more miles than Sara does, so that's the number of miles Ron gains on Sara.

The final step is identical to that of the train example above. We want to know when Ron will make up 5 miles, and he makes up 10 miles per hour. Given that information, Ron will catch up in a half hour.